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MULTI-MODE ANALYSIS OF RAYLEIGH-TYPE *Lg*. PART 1. THEORY AND APPLICABILITY OF THE METHOD

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ABSTRACT

Rayleigh-type *Lg* propagating in a laterally homogeneous continental crust can be synthesized by adding only a few overtones at periods greater than 2 sec. Under minimal assumptions, we show that wavenumber analysis of *Lg* recorded on a several hundred kilometers long linear array of 10 stations allow us to isolate the different overtones, providing a tool to study crustal structures and excitation of the overtones at the source. In this first paper, we use synthetic *Lg* seismograms to investigate the applicability of a time-frequency-wavenumber analysis technique (UC diagram algorithm) to realistic arrays of stations. The behavior of the algorithm in the presence of lateral heterogeneities is studied numerically by introducing either random or coherent phase perturbations. We find that (1) the method is tractable if random phase fluctuations from station to station are spread over less than half a cycle, and (2) coherent velocity changes between two halves of a profile are spatially averaged if they are too small to be resolved by the array.

INTRODUCTION

The *Lg* phase has been first described by Press and Ewing (1952) as a transverse impulsive phase propagating over several thousand kilometers across North America at crustal *S* velocities. For paths greater than 1000 km, *Lg* is a distinct phase while it merges with *Sg* at shorter distances. Significant radial and vertical components of *Lg* were also recognized soon thereafter (Lehmann, 1953; Gutenberg, 1955). Båth (1954, 1956) called Lg_1 and Lg_2 groups propagating, respectively, at 3.5 and 3.3 km/sec, and pointed out the complexity of this phase.

Early explanations involving rays guided in crustal low-velocity channels (Gutenberg, 1955) are no longer necessary since normal mode theory has shown that Love and Rayleigh modes propagating in any realistic continental structure exhibit group velocity extrema at periods smaller than 10 sec, which can be associated with *Lg* arrivals (Oliver and Ewing, 1957, 1958; Kovach and Anderson, 1964). It has been demonstrated more recently that interpretation of *Lg* in terms of higher modes is valid not only from the point of view of group velocities but also from the point of view of amplitude excitation at the source, mainly for mid-crustal earthquakes (for Love waves, see Knopoff *et al.*, 1974, and for Rayleigh waves, Panza and Calcagnile, 1975).

On the other hand, little progress has been made in observational analysis. Observations are usually very rough: existence or nonexistence of *Lg*, arrival time, peak-to-trough amplitude (Ruzaikin *et al.*, 1977; Chinn *et al.*, 1980). Explanation of the full observed *Lg* record is far from being achieved. For example, the *Lg* wave train does not terminate at group velocities of 3.1 to 3.2 km/sec, as predicted from normal mode theory for realistic models of continental crust. As pointed out by Ruzaikin *et al.* (1977) this probably represents direct evidence of scattering, which is a source of obvious difficulty in modeling the signal.

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In this paper, our approach is to analyze *Lg* by using a phase velocity criterion and wavenumber analysis. We shall show that stacking methods applied to a linear array of stations several hundred kilometers long should allow us to isolate the first few higher Rayleigh modes in the period range 2 to 5 sec provided that the crust is sufficiently homogeneous laterally. In a companion paper (Part 2), we will apply such methods to *Lg* recorded on the SCARLET array through the CEDAR system in southern California, and on the CALNET array in central California.

Preliminary Conditions for Multi-mode Analysis

The Fourier transform of a continuous signal spatially sampled by a linear array of stations can be expressed as the convolution product of the spatial Fourier transform of the signal with the "array response"

$$r(k) = \sum_{n=1}^{N} \exp \imath k x_n,$$

where k is the wavenumber and x_n, the spatial coordinates of station n. The basic wavenumber resolution of a linear array, δk, depends on the total length L of the array ($\delta k \sim 2\pi/L$). For equally spaced stations, $r(k)$ is a periodic function of period Δk which is inversely proportional to the spacing e between stations ($\Delta k = 2\pi/e$). Such a periodicity, or pseudoperiodicity in case of nonequally spaced stations, is responsible for spatial aliasing.

Spatial aliasing. Spatial aliasing occurs if the wavenumber spectrum is spread over an interval wider than Δk. Successful retrieval of the actual wavenumber spectrum of a continuous signal from Fourier analysis of observations made at discrete points in space thus requires some *a priori* knowledge of the wavenumber content of the signal. For a time signal, this problem is solved simply by use of analog antialiasing filters but no such device is of course possible for arbitrary space signals.

In the case of a pure seismic signal propagating in a laterally homogeneous structure, it is easy to determine the wavenumber content of the signal from normal mode theory. In particular, at a fixed period and within a narrow group-velocity window only a few modes have to be considered if the period is long enough. This is the concept applied by Cara (1976) and Nolet and Panza (1976) to the analysis of *Sa* at periods longer than 20 sec. The *Lg* situation is illustrated on Figure 1 which displays the phase and group velocity curves for Rayleigh modes propagating in a flat realistic model of the crust in southern California (Kanamori and Hadley, 1975). For periods greater than 2 sec, fewer than five overtones are expected to arrive in the group velocity window 3.2 to 3.6 km/sec. In the case of such pure multi-mode *Lg*, the wavenumber spectrum is restricted to a narrow band. For example, in the group velocity window 3.2 to 3.6 km/sec and at a period of 2.5 sec, the slowest and fastest phase velocities in Figure 1 are found for the first and fourth overtones, respectively. To avoid aliasing, the wavenumber periodicity Δk must then be greater than $k_4 - k_1$ and the critical spacing between station is $e_c = 2\pi(k_4 - k_1) \sim 55$ km, where $k_n = 2\pi/C_nT$ and C_n is the phase velocity of the nth overtone at period T. Note that we do not take the fundamental mode (*Rg* phase) into account here, because its group velocity at periods of a few seconds is very low in the presence of low-velocity superficial layers. Furthermore, the great sensitivity of that mode to low Q, highly heterogeneous shallow structure causes it to be strongly attenuated at epicentral distances of a few hundred kilometers.

The value of e_c thus estimated is slightly model dependent, but crust and upper mantle structure is known well enough that e_c cannot differ greatly from the above value. e_c is also period dependent but it is easy to show from Figure 1 that 55 km is an approximate lower bound in the group velocity window 3.2 to 3.6 km/sec and at periods between 2 and 5 sec.

Now, if some signal or noise with a different wavenumber spectrum arrives in the above group-velocity window—e.g., any signal arriving from a different azimuth in case of noncylindrical symmetry—the above conclusions are no longer valid and spatial aliasing can be a source of problems. The applicability of wavenumber analysis of *Lg* to arrays with such wide spacing between stations is thus *a priori* limited to circumstances where the signal-to-noise ratio is high, particularly where signal-generated noise due, e.g., to multipathing, is concerned.

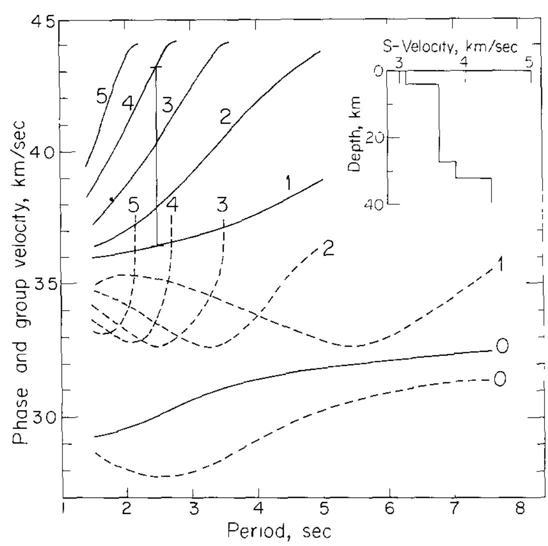

FIG. 1. Dispersion curves of Rayleigh overtones 1 to 5 computed for a realistic model of southern California. Dashed line, group velocity, solid line, phase velocity. The *S*-velocity model in the *inset* is derived from the *P*-velocity model of Kanamori and Hadley (1975) by setting Poisson's ratio to 0 25

Resolution. In order to isolate the different modes interfering in *Lg* through wavenumber analysis, we have to use a long enough array: δk must be smaller than the wavenumber spacing between modes. For the model used in Figure 1, the array length must be at least 225 km at 2.5 sec and 500 km at 1.5 sec but in contrast to the critical spacing e_c, this estimate is more strongly model dependent. For instance, if a low-velocity layer exists in the crust, the phase velocity curves can present kinks (Andrianova *et al.*, 1967; Panza *et al.*, 1972) and come so close to each other at some periods that an unrealistic array length would be required to isolate the modes.

A low-velocity channel might be present near 15 km in southern California as suggested, e.g., by fundamental Rayleigh mode data (Hadley and Kanamori, 1979). However, numerical experiments have shown that such phase velocity kinks are not likely to occur between 2 and 5 sec but might be found at periods smaller than 2 sec.

We can thus state the following preliminary set of conditions for multi-mode analysis of Rayleigh-type *Lg* in the period range 2 to 5 sec using linear receiver arrays: the total length of the array must be 200 to 500 km and the spacing between the stations smaller than 50 km. A 300-km-long array of 10 stations thus appears to be adequate for this purpose.

UC Diagrams

Several wavenumber analysis techniques have been developed for the study of multi-mode surface waves. To our knowledge, they have only been applied successfully to the *Sa* phase at periods greater than 20 to 30 sec (Nolet, 1975; Cara, 1978). A simple stacking algorithm, the UC diagram technique (Cara, 1976, 1978) seems appropriate for *Lg* in a first study because it needs only little *a priori* information about the signal. In fact, one needs only an approximate idea of the wavenumber content of the signal (total bandwidth, spacing between modes) to fix the geometry of the array as discussed previously, and an estimate for the central group velocity of the multi-mode wavepacket which is used in this algorithm to stack the records (Cara, 1976). Other techniques, such as band-pass wavenumber filtering to isolate a mode once its "subarray" phase velocity is known (Cara, 1978), require more information about the signal and could be used for amplitude and source excitation studies, but only after it has been recognized on UC diagrams that clear pure mode signals exist in the records.

The basic idea underlying the UC diagram algorithm is to stack N multi-mode records after narrow-band filtering and phase shifting in order to insure constructive interference of individual modes. Let $F(x_n, \omega)$ be the time Fourier transform of a record $f(x_n, t)$ observed at epicentral distance x_n, and let

$$G(k, \omega) = \sum_{n=1}^{N} F(x_n, \omega) \exp[-\alpha(\omega - \omega_0)^2] \exp[-i\, K(\omega)x_n]$$

where

$$K(\omega) = k + (\omega - \omega_0)/U_c.$$

In these expressions, ω_0 is the circular frequency at which analysis is to be performed and α controls the width of the Gaussian frequency filter. U_c is an input parameter, chosen equal to the central group velocity of the multi-mode wave packet and U_c^{-1} is approximately the first derivative of $K(\omega)$ at ω_0—in this paper we choose $U_c \approx 3.3$ km/sec. The modulus of the inverse Fourier transform $g(k, t)$ of $G(k, \omega)$ exhibits peaks at $k_p = \omega_0/C_p$ and $t_p = \bar{x}(U_p^{-1} - U_c^{-1})$ where C_p and U_p are, respectively, the phase and group velocity of mode p and where $\bar{x} = \sum_{n=1}^{N} x_n/N$ (Cara, 1976). The UC diagram technique consists in producing a set of contour plots of $|g(k, t)|$, conveniently plotted in terms of the hyperbolic coordinates $c = \omega_0/k$, and $U = \bar{x}/(t + \bar{x}/U_c)$, for a sequence of values of ω_0. Should dispersion properties vary slightly with epicentral distance along the profile, then it is important to note that U represents the group velocity between the source and the center of the array, and C is the phase velocity along the array (Cara, 1977). Being computed for a given, fixed period, a UC diagram contains information about the average dispersion properties of the wave train in a narrow-frequency interval. In particular, peak shapes in the U direction depend mainly on the frequency filter parameter α. In this paper, we chose α such that the total bandwidth of the Gaussian filter at -30 dB be $\omega_0/2\pi$.

For purposes of illustration we computed a set of synthetic *Lg* trains (Figure 2) along a 351-km-long array of nonequally spaced stations. We used the dispersion curves shown on Figure 1 and, based on our previous discussion, only included the fundamental Rayleigh mode and five overtones in the synthetics. These modes have been summed assuming a very simple constant unit amplitude for periods longer than 2 sec for each mode (0.5 for the fundamental) and a linear amplitude variation between 0.5 and 2 sec. The signals have also been convolved with a standard seismometer-galvanometer response ($T_s = T_g = 1$ sec, $\alpha_s = \alpha_g = 1$).

Figure 3 depicts a typical UC diagram calculated from this set of synthetic records at a period of 2.5 sec. The first three overtones are clearly identifiable, but the fourth one is already strongly dispersed at this period and does not generate a peak. Residual interference between the second and third overtones is probably responsible for the biased location of the third peak. Mode amplitudes should be computed for realistic source models but this is beyond the scope of this paper: our intent here is to show that modes can be isolated from *Lg* in a situation where interference

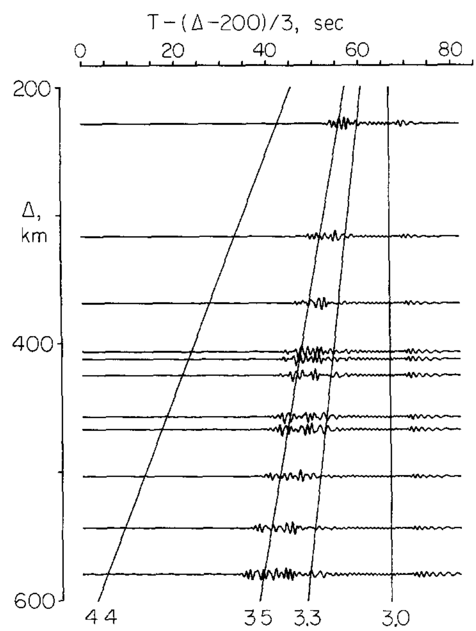

FIG. 2. Synthetic *Lg* and *Rg* phases computed by summing the first six Rayleigh modes with dispersion curves as shown in Figure 1, along a realistic array of stations. (Station distribution corresponds to a quasi-linear subarray of SCARLET.)

effects between modes play a major role. Choosing the same amplitude for all overtones can be considered as an attempt to model the worst situation we might have to face for this purpose. If one mode is more strongly excited than the others, it will be of course easier to isolate it from *Lg* records.

Mode identification from actual data. Processing records through the UC diagram technique yields a set of peaks in the UC plane at different periods. If for some reason one or several modes are weak or absent in the signal (due to weak mode excitation at the source, attenuation along the path, etc.), the identification of an observed peak as a mode of known rank is not obvious.

Figure 4 displays, e.g., the variation of overtone phase velocity for two models. The model shown in Figure 1 (primed numbers in Figure 4) exhibits a "normal" sub-Moho S velocity (4.5 km/sec) and the depth of the Moho is 32 km. The second model (unprimed numbers in Figure 4) has a homogeneous 3.5 km/sec S velocity in the crust, a Moho at 32 km and a low S velocity upper mantle ($V_s = 4.2$ km/sec). If the total sequence of modes is not present on an actual UC diagram, it is clear from

Figure 4 that identification of a peak with a mode of given rank can be a problem in cases where *a priori* estimates of model parameters are uncertain. On the other hand, if crustal parameters are known *a priori* with sufficient accuracy, mode identification of observed peaks should not be a problem for periods greater than 2 or 3 sec.

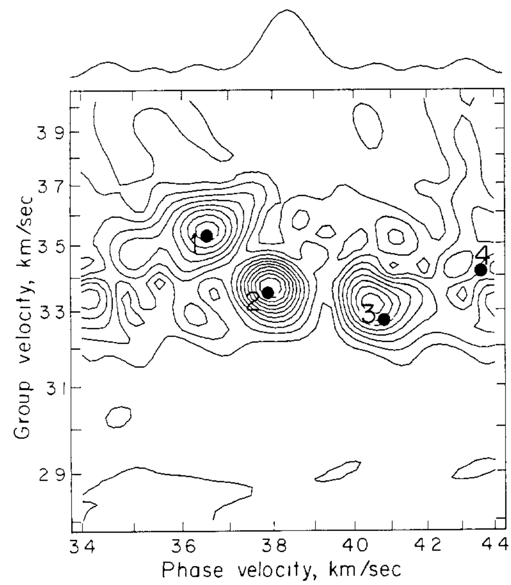

FIG 3. UC diagram after stacking the synthetic records of Figure 2 (period 2 5 sec, contour spacing 10 per cent of maximum amplitude) The dots show the theoretical values for phase and group velocities The vertical coordinate varies linearly with $1/U$ (time scale) and the horizontal coordinate with $1/C$ (wavenumber scale). The modulus of the array response $r(k)$ is shown on *top*.

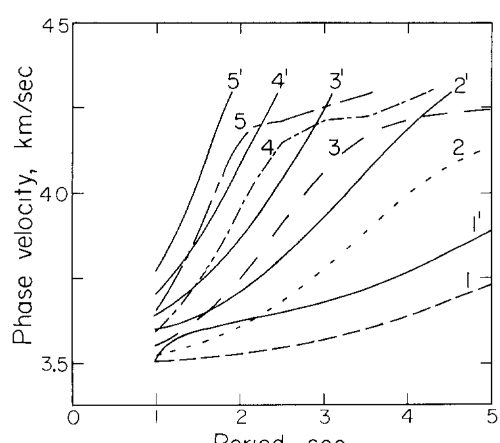

FIG 4 Phase velocity curves of the first five Rayleigh overtones computed for two models. Solid lines (primed numbers) correspond to the model shown in Figure 1 and dashed lines (unprimed numbers) to a one layer crust—low upper mantle velocity model (see text)

THE EFFECT OF PHASE PERTURBATIONS

In order to isolate Rayleigh overtones near 2.5-sec period from *Lg* records, we require arrays over several hundred kilometers long. We have assumed implicitly so far that lateral heterogeneities are weak enough that they do not invalidate the multi-mode representation of *Lg*. This is a rather strong assumption for these short-period crustal waves, and it is desirable to fix quantitative limits on permissible departures from the ideal, laterally homogeneous case.

For our purposes, the main effect of inhomogeneities is to alter partially or totally the constructive interference sought in UC diagrams. In what follows, we define phase perturbation as any change in the phase of a mode due to departure from a laterally homogeneous structure. Modeling overtone propagation for realistic laterally varying structures, including topography, is a very difficult task which lies well beyond the scope of the present paper. However, order of magnitude effects can be estimated in two especially simple cases. They are: (1) random fluctuations (statistically independent from station to station); and (2) coherent phase perturbations within a subgroup of stations. We attempt to quantify those effects here.

Let us first note that, irrespective of the cause of phase perturbations, for a fixed overtone n, at a fixed period T, the apparent phase at an epicentral distance x_i is of the form $\phi_n(x_i, T) + 2\pi\delta t_n(x_i, T)/T$. Here, ϕ_n is the phase calculated for the unperturbed model (e.g., a spatially averaged structure for the area), and δt_n is a time term which accounts for the local phase perturbation. In a multi-mode signal, δt_n depends *a priori* on n, since different modes sample the structure differently and thus are not sensitive to the same heterogeneities. A more precise statement would necessitate considerable analysis, but is fortunately not required for our purposes. We have perturbed synthetic *Lg* records under the simplifying assumptions that (1) δt_n is independent of n and (2) the mode amplitudes are not affected, and have submitted the resulting profiles to UC analysis. In that case, perturbations reduce to simple time shifts of the multi-mode wave train if one looks at a single period.

The two types of perturbations discussed above (random and coherent) are then modeled, respectively, through the two relations

$$\delta t_n(x_i, T) = \Delta t \,\mathrm{rand}(i)$$

or

$$\delta t_n(x_i, T) = (\epsilon/V)(x_i - x_p), \quad x_i \geqq x_p$$

where rand(i) is a random number with uniform probability on [−0.5, 0.5], Δt a scaling parameter, V, a reference velocity, ϵ, a relative velocity offset for the second portion of the profile.

Random fluctuations. Figures 5 and 6 correspond to two sets of random numbers at $\Delta t = 1$ sec. The UC coordinates of overtone peaks remain very stable in this case, but spurious peaks are sometimes generated (Figure 6). Further numerical experiments with Δt ranging from 0.5 to 1.5 sec and other sets of random time shifts show that apparent phase velocities for modes 1 and 2 were perturbed by less than 0.02 km/sec but that the peak amplitudes could be strongly affected, particularly for the larger values of Δt. Stronger perturbations were obtained for mode 3 (± 0.06 km/sec). It is interesting that the unperturbed profile already yields a slightly biased estimate of this peak due to a lack of wavenumber resolution, as mentioned earlier.

Figure 7 exhibits a much higher perturbation level for $\Delta t = 2.5$ sec, which coincides with the period of the UC diagram. In this particular example, peaks are still recognizable at nearly correct locations, but many spurious peaks have been generated too. In fact, other sets of random numbers lead to such changes in the topography of the diagram that overtones are not recognizable.

Although this numerical experiment is based on an oversimplification, it clearly indicates that the UC diagram technique remains stable under phase fluctuations with a random spatial distribution, and up to the order of half a cycle in magnitude. However, these fluctuations do seem to affect peak amplitudes by a large amount,

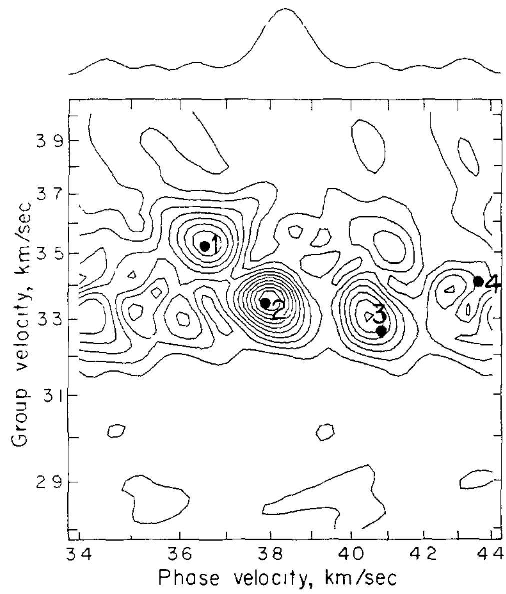

FIG. 5 UC diagram obtained for synthetic records simulating the propagation of *Lg* on the profile shown in Figure 2, with randomly distributed time shifts spread over a 1-sec interval.

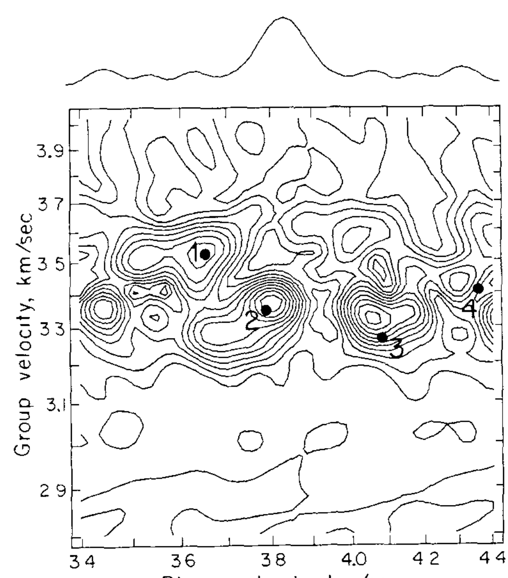

FIG. 6. Same as Figure 5 for a different distribution of random time shifts

so that even in cases where subarray dispersion measurements can be achieved successfully, great caution must be exercised before interpreting peak amplitudes (in terms of overtone excitation at the source, for example).

Coherent fluctuations. In a second numerical experiment, the same set of synthetic

Lg records were left unperturbed for epicentral distances shorter than 412 km, and a time shift proportional to distance was applied between 412 and 580 km. This mimics the effect of a relative perturbation in the phase velocity of the form

$$\Delta C/C = -\epsilon\, C/V.$$

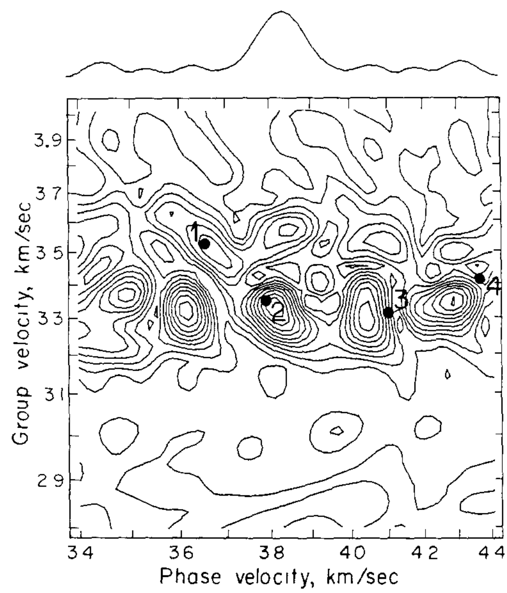

FIG 7 Same as Figure 5 for time shifts spread over a 2 5-sec interval.

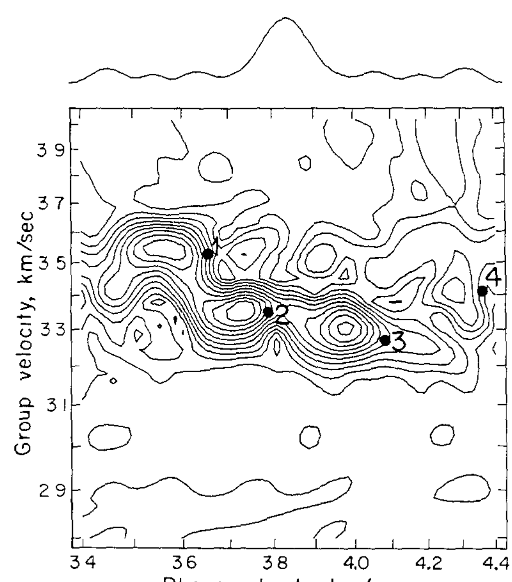

FIG 8 Same as Figure 5 for coherent time shifts simulating a −3.25 per cent decrease of a 3.9 km/sec-phase velocity on the second half of the profile

Figures 8 and 9 show UC diagrams for $\epsilon = 2.5 \times 10^{-2}$ and 5×10^{-2}, respectively, and $V = 3$ km/sec (phase velocity variations of −3.25 and −6.5 per cent for $C \simeq 3.9$ km/sec, the central phase velocity for these UC diagrams).

On Figure 8, longitudinal variations of about 3 per cent in phase velocity (depending on the mode) broaden the peaks and shift them toward lower phase velocities, as expected. Stronger variations of about 6 per cent (Figure 9) lead to clear splitting of the peaks for modes 1 and 2. For these two modes we obtain therefore two phase velocity estimates which correspond each to a portion of the profile. Thus it seems that UC peak splitting results when coherent phase velocity changes reach about 5 per cent along half the profile. Note that this corresponds to the typical phase velocity spacing of modes near 2.5-sec period. This is also the intrinsic resolution of the array, because of the way it was constructed *a priori* for overtone separation.

These two numerical experiments permit us to place quantitative limits on the applicability of the method. It yields stable results, averaging possible phase fluctuations along the station array, provided that (1) statistically independent random phase fluctuations from station to station have a spread of less than half a cycle, and (2) coherent velocity variations between the two halves of the profile are less

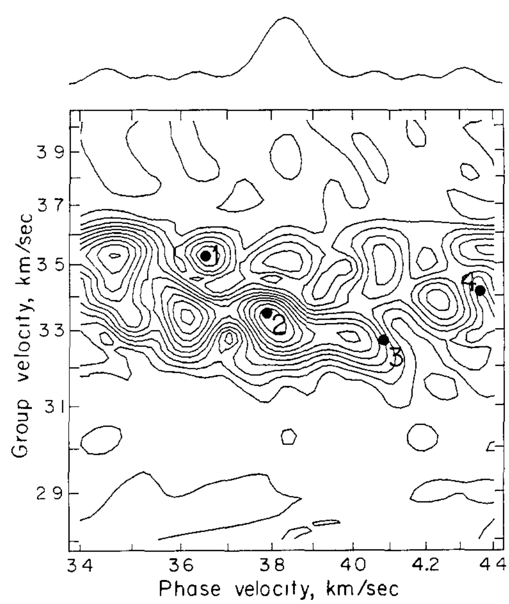

FIG 9 Same as Figure 8 for a −6 5% decrease of velocity

than the resolution of the array. When these conditions are violated, spurious peaks can be generated and peak splitting can occur. Of course, in this case, because the wavenumber resolution of the array is about the same as the wavenumber spacing between modes, a representation of the *Lg* wave field in terms of overtones with coherent propagation over several hundred kilometers loses all significance.

Finally, we must add that the critical conditions described in this section touch on problems of a different nature than the problems discussed in detail by Nolet and Panza (1976). These conditions are more directly related to the physics of wave propagation on actual structures than to the geometrical details of the array. It is also interesting to point out that for the *Sa* phase, another type of multi-mode wave of longer period, the maximum observed lateral variations for the first Rayleigh overtone reach 0.2 km/sec at 50-sec period (Cara *et al.*, 1980) while the phase velocity spacing between first, second, and third overtones is greater than 0.6 km/sec for realistic earth models. This is probably the reason why stacking methods such as the UC diagram technique work well for long-period overtones even in the case of strong lateral upper mantle heterogeneities, as is the case across the whole continental United States WWSSN network for example (Cara, 1978).

Conclusions

This short study shows that although severe conditions are required for retrieving modes from *Lg* records, this is theoretically possible in the period range 2 to 5 sec, even at a few hundred kilometers from the source, by using linear 300- to 500-km-long arrays of about 10 stations. The more severe limitation by far seems to be that the crust be rather homogeneous laterally in the area under study.

Lateral arrivals of any kind can modify the wavenumber content of the signal. In this case, the low station density proposed here becomes a source of serious difficulties because of spatial aliasing. Another source of difficulty might be the mode identification from observed UC diagrams unless the crust is known with sufficient accuracy in the area under study to predict overtone phase velocities close enough to actual ones.

In the presence of lateral variation of structure under the array, the apparent phase of *Lg* signals can differ locally from the values predicted by a standard multi-mode representation of the wave field. A numerical experiment involving random as well as coherent phase perturbations shows that the UC diagram technique yields stable estimates of spatially averaged phase velocities for the different overtones, provided that (1) random phase fluctuations from station to station remain smaller than half a cycle, or (2) apparent velocity variations between two halves of a profile remain less than the resolution of the array and thus less than the phase velocity spacing between modes (e.g., ~5 per cent at 2.5-sec period).

If such conditions are satisfied, multi-mode analysis of *Lg* on arrays of stations should provide a powerful tool to study details in the crustal structure and crustal overtone excitation at the source. Actual applications to data recorded on the SCARLET array through the CEDAR system and on the CALNET array in central California are described in a companion paper.

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